

sign equations,

$$\frac{\sqrt{3} \cdot 1.84}{\pi} \frac{\psi}{\sqrt{\epsilon_f}} \frac{\sqrt{\mu_{eff}}}{k/\mu} = 1, \quad (7)$$

is not easily satisfied when operating below resonance in  $L$  or  $S$  band. In fact, since  $\sqrt{3} \cdot 1.84/\pi \approx 1$  and  $\psi \leq 1$  which is the angle defined by Fig. 2, the quantity

$$\frac{1}{\sqrt{\epsilon_f}} \frac{\sqrt{\mu_{eff}}}{k/\mu}$$

must equal or exceed unity to satisfy (7). At low frequencies, where  $4\pi M_s/H_0$  cannot be much smaller than one, this may be impossible. See Fig. 1. At higher frequencies it is inconvenient to make the quantity  $(1/\sqrt{\epsilon_f})(\sqrt{\mu_{eff}}/k/\mu)$  approximate unity because this would require a rather small  $K/\mu$ . Small values of  $K/\mu$  reduce the bandwidth of the circulator. The graph in Fig. 1 shows how  $\mu_{eff}$ ,  $k/\mu$ , and  $\sqrt{\mu_{eff}}/k/\mu$  vary with  $H_i/H_0$  for three particular values of  $4\pi M_s/H_0$ . Thus, instead of using the two design equations of Bosma,<sup>1</sup> (7), and

$$\frac{R}{\lambda} = \frac{1.84}{2\pi\sqrt{\mu_{eff}\epsilon_f}} \quad (8)$$

and solving them to find  $H_i$  and the radius of the ferrite disc  $R$ , the design can be made in the following way:

- 1) Choose  $H_i$  in accordance to (5).
- 2) Calculate  $\mu_{eff}$  from (3).
- 3) Find  $R$  from (8).
- 4) Match the ports of the circulator to the 50 ohm external line.

The impedance-transforming arrangement should have as wide a frequency band as possible so as not to limit the bandwidth of the circulator. The use of a half wave tapered line transformer and dielectric discs (Fig. 2) of high dielectric constant  $\epsilon_d$  gives a rather compact solution. In this case  $\epsilon_d$  must satisfy the relation

$$\sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{\sqrt{\epsilon_d}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\sqrt{3} \cdot 1.84}{\pi} \psi \sqrt{\frac{\mu_{eff}}{\epsilon_f}} \frac{k}{\mu} \quad (9)$$

where  $\sqrt{\mu_0/\epsilon_0} = 120\pi$  is the intrinsic impedance of free space. The use of  $\lambda/4$  transformers offers another solution. In this case the dielectric constant of the dielectric disks must satisfy the relation

$$\left[ \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\sqrt{3} \cdot 1.84}{\pi} \psi \sqrt{\frac{\mu_{eff}}{\epsilon_f}} \frac{k}{\mu} \right] \sqrt{\frac{\mu_0}{\epsilon_0}} = \left( \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{\sqrt{\epsilon_d}} \right)^2 \quad (10)$$

The width of the strip line and the distance between the ground plates must be such that the characteristic impedance of the strip line is 50 ohms when air insulated.

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## Surface Wave on a Perfectly Conducting Plane Covered with Magnetoplasma

It has been found by Ishimaru<sup>1</sup> and Seshadri<sup>2</sup> that a unidirectional TEM surface wave is trapped along a perfectly conducting plane covered with *transversely* magnetized semi-infinite plasma.

This communication gives a brief description of a surface wave which is found to be trapped along a perfectly conducting plane covered with *longitudinally* magnetized semi-infinite plasma. When the plasma is magnetized in the direction of propagation, the electromagnetic fields can not be separated into TE or TM modes, and are given by a combination of two characteristic wave modes having each different, but interrelated, characteristic values to satisfy Maxwell's equations for magnetoplasma. The combination is determined by the boundary conditions.

The coordinate is chosen so that the perfectly conducting plane is located in the  $y$ - $z$  plane ( $x=0$ ) and the  $z$  axis is the direction of propagating waves or the direction of magnetization.

All the field components are assumed to be independent of the  $y$  coordinate and have the time and  $z$  dependence of  $e^{j(\omega t - k_0 z)}$  ( $k_0$ : relative axial propagation constant). The electromagnetic fields satisfying Maxwell's equations and the boundary conditions can be expressed by a combination of a right-handed plane wave mode and a left-handed plane wave mode with the same axial propagation constant  $k_0$ . The resulting field equations are given by

$$\left. \begin{aligned} E_x &= Z_0(s_1^2 + \epsilon_0) \left\{ (s_1^2 - h^2 + \epsilon_1) e^{-s_1 k_0 x} - (s_2^2 - h^2 + \epsilon_1) e^{-s_2 k_0 x} \right\} \\ E_y &= -j\epsilon_0 Z_0(s_1^2 + \epsilon_0) \left\{ e^{-s_1 k_0 x} - e^{-s_2 k_0 x} \right\} \\ E_z &= jZ_0 s_1 h(s_1^2 - h^2 + \epsilon_1) \left\{ e^{-s_1 k_0 x} - e^{-s_2 k_0 x} \right\} \\ H_x &= j\epsilon_0 h(s_1^2 + \epsilon_0) \left\{ e^{-s_1 k_0 x} - e^{-s_2 k_0 x} \right\} \\ H_y &= \epsilon_0 h(s_1^2 - h^2 + \epsilon_1) \left\{ e^{-s_1 k_0 x} - \frac{s_1}{s_2} e^{-s_2 k_0 x} \right\} \\ H_z &= -\epsilon_0(s_1^2 + \epsilon_0) \left\{ s_1 e^{-s_1 k_0 x} - s_2 e^{-s_2 k_0 x} \right\} \end{aligned} \right\} \quad (1)$$

with the dispersion equation

$$s_1^2 + s_2^2 + s_1 s_2 - h^2 + \frac{\epsilon_1^2 - \epsilon_2^2}{\epsilon_1} = 0, \quad (2)$$

where  $s_1$  and  $s_2$  are relative transverse propagation constants normalized with respect to free space propagation constant  $k_0$  and are solutions of the quadratic equation:

$$\begin{aligned} s^4 - s^2 \left[ h^2 \left( 1 + \frac{\epsilon_3}{\epsilon_1} \right) + \frac{\epsilon_2^2}{\epsilon_1} - \epsilon_1 - \epsilon_3 \right] \\ + \frac{\epsilon}{\epsilon_1} [(h^2 - \epsilon_1)^2 - \epsilon_2^2] = 0. \end{aligned} \quad (3)$$

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<sup>1</sup> A. Ishimaru, "The effect of a unidirectional surface wave along a perfectly conducting plane on the radiation from a plasma sheath," presented at the 2nd Symp. on the Plasma Sheath, Boston, Mass.; April, 1962.

<sup>2</sup> S. R. Seshadri, "Excitation of surface waves on a perfectly conducting screen covered with anisotropic plasma," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 573-578; November, 1962.

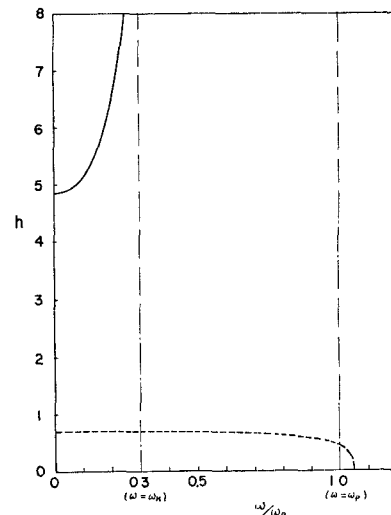


Fig. 1—Frequency characteristics of the relative propagation constants for  $\omega_H < \omega_p$ , — : trapped wave; --- : plane wave; - - - : improper wave.

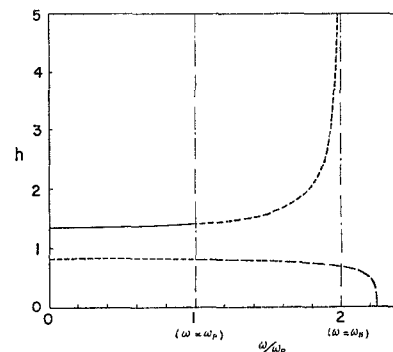


Fig. 2—Frequency characteristics of the relative propagation constants for  $\omega_H > \omega_p$ , — : trapped wave; --- : plane wave; - - - : improper wave

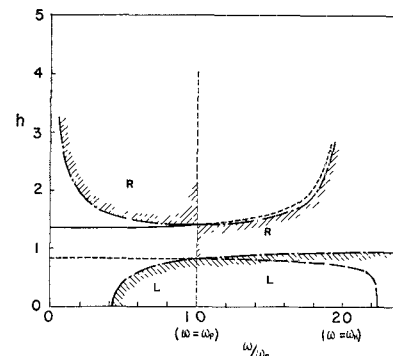


Fig. 3—Illustrating the relation between the propagation constant  $h$  of the trapped wave and the permissive axial propagation constants of two characteristic plane waves in a free magnetoplasma. Symbols  $R$  and  $L$  denote the regions of the permissive axial propagation constants of a nonattenuating right-handed and a left-handed plane wave, respectively.

Solving (2) and (3) for  $h^2$  and  $s_{1,2}^2$  leads to

$$h^2 = \epsilon_1 \pm \sqrt{\epsilon_1(\epsilon_1 - 1)}, \quad (4)$$

and

$$s_{1,2}^2 = \frac{1}{2\epsilon_1} [(\epsilon_1 + \epsilon_3)(h^2 - \epsilon_1) + \epsilon_2^2 \pm \epsilon_2 \sqrt{2(\epsilon_1 + \epsilon_3)h^2 - \epsilon_1 + \epsilon_3}]. \quad (5)$$

For  $\omega$  smaller than  $\sqrt{\omega_p^2 + \omega_H^2}$ ,  $h^2$  given by (4) is always real, and  $s_1 s_2$  is also real from (2) and (3). For this case the signs ( $\pm$ ) in (4) corresponds to  $\epsilon_1 \epsilon_3 \leq 0$  if  $s_1 s_2 > 0$ , and  $\epsilon_1 \epsilon_3 \geq 0$  if  $s_1 s_2 < 0$ , respectively.

When  $s_1 s_2$  is negative, real part of either of  $s_1$  and  $s_2$  is negative in general, therefore the fields increase to infinity at  $x \rightarrow \infty$ . In Figs. 1 and 2 the frequency characteristics of the relative propagation constants for these improper modes are plotted by the dotted lines.

When  $s_1 s_2$  is positive, and in the frequency range of

$$\omega_p < \omega < \sqrt{\omega_p^2 + \omega_H^2}, \quad (6)$$

either of  $s_1$  and  $s_2$  is a positive imaginary and the other is a negative imaginary. This implies that the fields are merely the superposition of two nonattenuating plane waves, a left-handed and a right-handed plane wave, one being incident on the conducting plane and the other reflecting from the plane. The angle of incidence is different from that of reflection since  $|s_1| \neq |s_2|$ . The frequency characteristics of  $h$  are plotted by dashed lines in Figs. 1 and 2.

In the frequency range of

$$\omega < \text{Min}(\omega_H, \omega_p), \quad (7)$$

the electromagnetic wave is found to be trapped along the conducting plane. The relative propagation constant  $h$  is given by  $[\epsilon_1 + \sqrt{\epsilon_1(\epsilon_1 - 1)}]^{1/2}$  and is plotted by the solid lines in Figs. 1 and 2. For this case  $s_1$  and  $s_2$  are both positive real numbers or conjugate complex numbers with positive reals, therefore the equiphase surface is vertical to the guiding plane. The group velocity  $v_g$  of this trapped wave is obtained as

$$c/v_g = h \left[ 1 + \left( 1 - \frac{1}{2h^2} \right) \frac{\omega_p \omega^2}{(\omega_H^2 - \omega^2)(\omega_p^2 + \omega_H^2 - \omega^2)^{1/2}} \right], \quad (8)$$

where  $c$  is the velocity of light.

Fig. 3 illustrates the relations between the propagation constant  $h$  discussed above and the permissive  $z$ -directed propagation constant of nonattenuating left-handed and right-handed plane waves in a free magnetoplasma. It is seen from this figure that there can exist no right-handed and left-handed plane waves propagating with the axial phase velocity equal to that of the trapped wave in the region of  $\omega < \text{Min}(\omega_p, \omega_H)$ . The incident and reflecting angles of the left handed and right-handed waves take imaginary values in the trapped-wave region.

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## A Multistub Coaxial Line Tuner

### INTRODUCTION

To date, one of the major difficulties in precision coaxial line reflectometer work has been the lack of suitable coaxial line tuners. The tuner described here was designed specifically for reflectometer work and overcomes the major problems of mechanical instability, coarse tuning adjustments, and leakage prevalent in presently available coaxial line tuners.

### DISCUSSION OF TUNER

The geometry of the tuner is essentially that of a parallel plane transmission line<sup>1</sup> where the ends of the parallel plates have been closed at a distance sufficiently far from the center of the line so as not to alter appreciably the characteristics of the parallel plane line.

The electric field configuration in this type line is shown in Fig. 1. The field is very strong in the narrow gap between the center conductor and the side wall and is very weak in the large region between the center conductor and the top. This condition is represented by the high concentration of lines in the narrow gap and low concentration of lines in the large region. To get the desired tuning operation, one set of tuning stubs has been placed in the region of the concentrated electric field and another set of tuning stubs has been placed at the top of the tuner for operation in the region of the weak field. The location of these tuning stubs is shown in Fig. 2. The stubs in the region of the concentrated electric field give a coarse tuning operation, and the stubs in the region of the weak field give a fine tuning operation.

The tuning stubs are threaded through a collet-type lock to obtain smooth stub adjustment and to prevent leakage to the out-

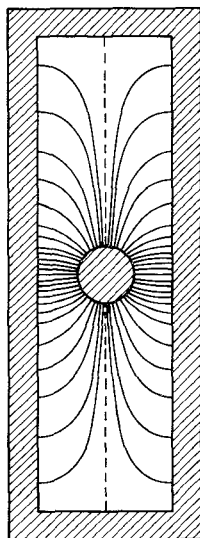


Fig. 1—Electric field configuration in the enclosed parallel plane line.

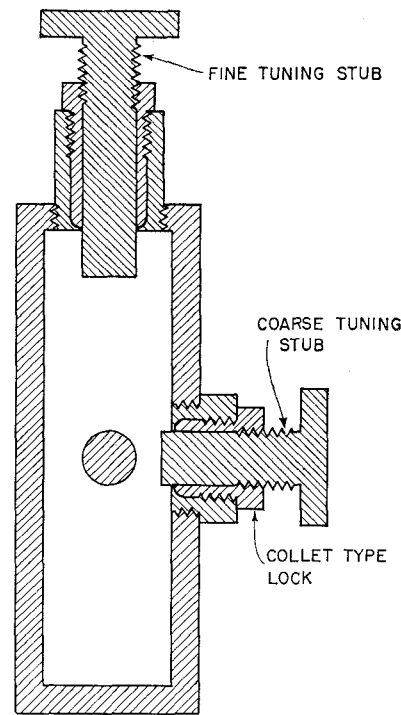
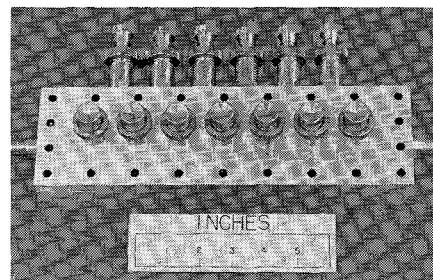
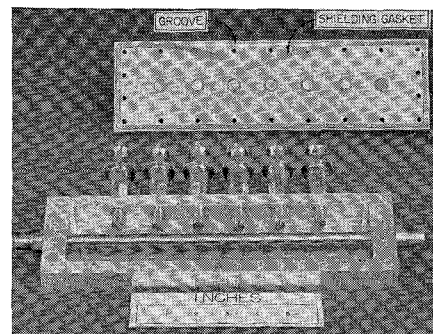


Fig. 2—Cross section of tuner showing location of tuning stubs and collet-type locks.



(a)



(b)

Fig. 3—(a) Photograph of tuner.  
(b) Photograph of tuner.

side of the line. Also, a groove has been cut completely around the side plate and a woven gasket has been placed in this groove to prevent any leakage from around the side plate. This groove is shown in Fig. 3.

### RESULTS

Fig. 3 shows a photograph of a tuner designed to tune out a reflection coefficient of 0.33 (VSWR of 2 to 1) over the frequency

Manuscript received April 28, 1964.  
<sup>1</sup> W. B. Wholey and W. N. Eldred, "A new type of slotted line section," *Proc. IRE*, vol. 38, pp. 244-249; March, 1950.